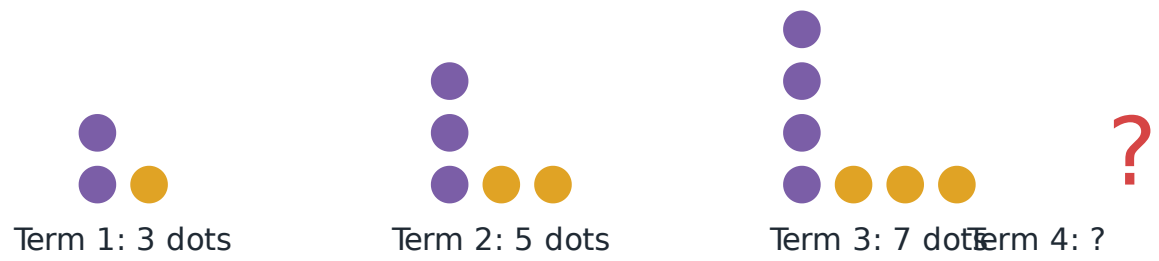
# Extension & Challenge — Patterns & Relationships

## Part A — Growing Patterns & Rules

Every growing pattern hides two rules. The **term-to-term** rule tells you how to jump from one term to the next. The more powerful **position-to-term** rule lets you find *any* term straight away, even the 100th, without drawing it.

> **Worked example.** A pattern goes 4, 7, 10, 13, …
>
> Term-to-term: $+3$ each time. To find the position-to-term rule, notice each term is $3 \times (\text{term number}) + 1$. Check term 2: $3 \times 2 + 1 = 7$. So the 20th term is $3 \times 20 + 1 = 61$.

### Growing pattern — how many dots next?




*An L-shaped growing pattern: 3, 5, 7, … dots.*

1 How many dots are in Term 4 and Term 5? Sketch Term 4 to check.

Term 4: _____ Term 5: _____

2 Describe the **term-to-term** rule (how you get from one term to the next) in words.

3 Find the **position-to-term** rule that connects the term number $n$ to the number of dots.

Number of dots = _____

4 Use your rule to predict the number of dots in the 10th term and the 50th term.

10th term: _____ 50th term: _____

5 Which term has exactly $99$ dots? Show how you worked backwards from the rule.

6 **Prove it.** Will any term in this pattern ever have an *even* number of dots? Explain your reasoning.

## Part B — Function Machines & Sequences

A **function machine** takes an input number, applies one or more operations in order, and produces an output. Reading a table of inputs and outputs lets us discover the hidden rule — and reversing the operations lets us work backwards.

> **Worked example.** A machine does $\times 4$ then $+2$. Input $3$ gives $3 \times 4 = 12$, then $12 + 2 = 14$.
>
> To work backwards from an output of $14$, undo in reverse: $14 - 2 = 12$, then $12 \div 4 = 3$.

1 A function machine does $\times 3$ then $-1$. Complete the output column.

| Input | Output |
|---|---|
| $1$ | |
| $2$ | |
| $5$ | |
| $10$ | |

2 Look at this input/output table and find the one-step or two-step rule.

| Input | Output |
|---|---|
| 2 | 9 |
| 3 | 13 |
| 5 | 21 |

Rule: _____

3 **Work backwards.** A machine does $\times 2$ then $+5$. Its output is $23$. What was the input?

4 **Open challenge.** Design a two-step function machine that turns $4$ into $20$. Then find a *different* two-step machine that also turns 4 into 20.