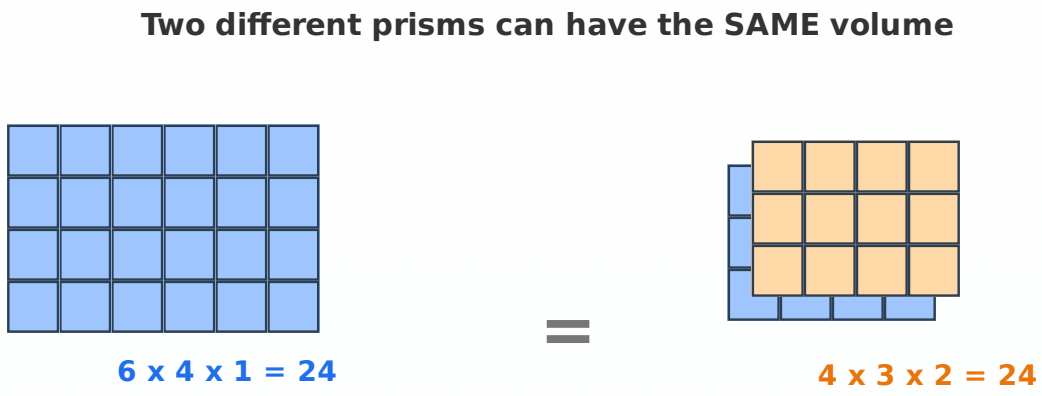
Investigation — Same Volume, Different Prisms

Investigation: building prisms with a fixed number of cubes

In this investigation you will build prisms by hand and record what you find. The big question: **can two prisms that look completely different have exactly the same volume?**



Both prisms use 24 cubes, yet they are different shapes.

You will need: 24 centimetre cubes (then 36 for the extension), and a partner to compare results with.

Build and record

Q1. Take exactly *24* cubes. Build as many different rectangular prisms as you can. Record each one in the table (the first row is done for you).

Length	Width	Height	Length × Width × Height	Volume
6	4	1	6 × 4 × 1	24

Q2. How many different prisms did you find altogether? Compare with a partner — did you find the same number? _____

Look for patterns

Q3. Look down your "Volume" column. What do you notice about every prism you built?

Q4. Did the prisms all look the same? Did they all have the same volume? Complete: *"Two prisms can have the same volume even when they have different..."* _____

Q5. **Reasoning.** Your partner says, "A taller prism must hold more cubes." Use one of your prisms to explain why this is not always true.

Q6. **Extend.** Now try 36 cubes. Predict how many different prisms you can build, then check by building them.

Answer key (teacher use)

Suggested answers and the complete list of prisms for each target volume.

> **Teacher answer key.** The point of the task is the pattern in Q3–Q5, not finding every prism. Treat rotations of the same prism (e.g. $6\times4\times1$ and $4\times6\times1$) as the same shape.

Q1. The distinct rectangular prisms with volume 24 are: $1\times1\times24$, $1\times2\times12$, $1\times3\times8$, $1\times4\times6$, $2\times2\times6$, $2\times3\times4$. Every row in the "Volume" column must equal 24.

Q2. Six distinct prisms (treating rotations as the same). Partners should agree on the count once duplicates that are just rotations are removed.

Q3. Every prism has a volume of 24 cubic units — the volume is always the same.

Q4. No, the prisms do not look the same, but they all have the same volume. Completion: "... different **dimensions (shapes)**."

Q5. Sample: the $2\times2\times6$ prism is tall but holds the same 24 cubes as the short, wide $1\times4\times6$ prism. Being taller is offset by being narrower, so height alone does not decide volume.

Q6. The distinct prisms with volume 36 are: $1\times1\times36$, $1\times2\times18$, $1\times3\times12$, $1\times4\times9$, $1\times6\times6$, $2\times2\times9$, $2\times3\times6$, $3\times3\times4$ — eight in total. Accept reasonable predictions; the check is that each set of dimensions multiplies to 36.